Rainfall depth-duration-frequency curves and their uncertainties

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Summary Rainfall depth-duration-frequency (DDF) curves describe rainfall depth as a function of duration for given return periods and are important for the design of hydraulic structures. This paper focuses on the effects of dependence between the maximum rainfalls for different durations on the estimation of DDF curves and the modelling of uncertainty of these curves. For this purpose the hourly rainfall depths from 12 stations in the Netherlands are analysed. The records of these stations are concatenated to one station-year record, since no geographical variation in extreme rainfall statistics could be found and the spatial dependence between the maximum rainfalls appears to be small. A generalised extreme value (GEV) distribution is fitted to the 514 annual rainfall maxima from the station-year record for durations of 1, 2, 4, 8, 12 and 24 h. Subsequently, the estimated GEV parameters are modelled as a function of duration to construct DDF curves, using the method of generalised least squares to account for the correlation between GEV parameters for different durations. A bootstrap estimate of the covariance matrix of the estimated GEV parameters is used in the generalised least squares procedure. It turns out that the shape parameter of the GEV distribution does not vary with duration. The bootstrap is also used to obtain 95%-confidence bands of the DDF curves. The bootstrap distribution of the estimated quantiles can be described by a lognormal distribution. The parameter $\sigma$ of this distribution (standard deviation of the underlying normal distribution) is modelled as a function of duration and return period.

Introduction

Statistics of extreme rainfall are important to society for (i) design purposes in water management — such as the construction of sewerage systems, determination of the required discharge capacity of channels and capacity of pumping stations — in order to prevent flooding, thereby
Rainfall depth-duration-frequency (DDF) curves are constructed to understand the behavior of extreme rainfall events. The first step in constructing DDF curves is to fit some theoretical distribution to the extreme rainfall amounts for a number of fixed durations. A logical step to proceed then is to describe the change of the parameters of the distribution with duration by a functional relation. From the fitted relationships, the rainfall depths for any duration and return period can be derived. A problem in this approach is that the estimated parameters for different durations are correlated. Standard regression techniques may then not be appropriate to estimate the unknown coefficients in the relationships that determine the DDF curves and the uncertainty of these relationships.

Though frequently used, the Gumbel distribution may underestimate quantiles for long return periods, leading to intensity-duration-frequency (IDF) curves that describe rainfall depth as a function of duration for given return periods or probabilities of exceedance. In particular for short durations, rainfall intensity has often been considered rather than rainfall depth, leading to IDDF curves. The method of derivation of the two types of curves is, however, identical.

Koutsoyiannis et al. (1998) present a mathematical framework for studying IDF relationships, which also applies to DDF curves. The bootstrap is used both for the estimation of correlation between estimated GEV parameters and for the confidence bands of the DDF curves. The latter shows similarities with the resampling technique presented by Burn (2003) to calculate confidence intervals for flood quantiles. Innovative aspect of this paper is that the uncertainty in rainfall DDF curves is described with a lognormal probability distribution.

This paper is organized as follows. First, the data are described. Next, the construction of the station-year record is justified. Subsequently, the GEV fits to the annual maxima of this record and the modeling of the change of the GEV parameters with duration are addressed. This is followed by a description of the construction of DDF curves and their uncertainties. The paper closes with a discussion and conclusions.

Rainfall data

Rain gauge networks

KNMI maintains two independent rain gauge networks: an automatic network of approximately 35 gauges (≈1 station per 1000 km²) and a manual network of approximately 325 gauges (≈1 station per 100 km²). Originally the automatic network was operated using mechanical pluviographs. These have been replaced by electronic rain gauges from the end of the 1970s. The electronic rain gauge measures the precipitation depth using the displacement of a float placed in a reservoir. The 24-h precipitation depth from the manual gauges is measured at 0800 UTC. Detailed information on the rain gauge networks of KNMI can be found in KNMI (2000).

To perform a reliable extreme value analysis, only stations with automatic rain gauges were selected for which at least 29 years of hourly precipitation depth data were available from the mid 1970s. It is noted that these depths are clock-hour sums. This selection resulted in a data set with time series from 13 stations distributed over the Netherlands. The 30-year record of one station was removed, since it was located only 7 km from De Bilt, for which a much longer record was available.

The locations of the selected stations are shown in Fig. 1 and are listed in Table 1. The time series, in total 514 station years, all end in 2005. If more than 5 days in a year were missing, the year was removed from the data set. In total only 3 station years were removed. For the automatic gauge in De Bilt, the annual 1-h and 2-h rainfall maxima based on continuous recording, so-called sliding maxima, are available for the period 1906–1990. The data from the manual network were only used for adjustment of the automatic gauge observations (see below). Until the mid 1990s all stations were equipped with a collocated manual gauge. In 2005, the distance between the selected automatic gauges and the nearest manual gauge ranged from 0.3 to 6.1 km.

Adjustment of automatic gauge data

The WMO (1981) Guide to Hydrological Practices states that it was decided that the standard non-recording raingauge
measurements should be the official rainfall readings at the station, and that a correction factor should be applied to hourly rainfall and maximum intensity data, based on the ratio of the daily total by standard gauge to the total by recording gauge.” Before 1982 the archived hourly sums from the automatic gauges were adjusted by default with the daily sums from the collocated manual gauge. From 1982 the annual rainfall sums from the manual gauges are on average 5% larger than those from the automatic gauges. To promote the homogeneity of the data set it was decided to adjust the remaining 56% of automatic gauge data (1982–2005) by the same procedure, so also the data from the mid 1990s using the readings from the nearest manual gauge.

Fig. 2 shows a typical scatter plot of the daily precipitation depths from the automatic and the manual gauges in De Bilt during 1987. Evidently the two gauges correspond rather well and the adjustment factors are generally close to unity. The extreme value analysis presented in the remainder of this paper has been carried out using the adjusted data set. When the same analysis is performed on the (partly) unadjusted data set the differences are small.

Regional variability in extreme rainfall statistics

The Netherlands has a temperate climate with mean annual rainfall varying from 768 to 848 mm for the 12 selected stations. This low variation is due to the absence of significant orography. Most daily (0800–0800 UTC) annual maxima occur in the period May to December, whereas most annual 1-h maxima occur from May to September, caused by the larger influence of convective rainfall in summer. In this section regional variability in extreme rainfall statistics is investigated. First the GEV distribution is introduced. Then the regional variability of its parameters is studied.

Fitting a GEV distribution

The GEV distribution has been used worldwide to model rainfall maxima, see e.g. Schaefer (1990), Alila (1999), Gelens (2002), Fowler and Kilsby (2003) and Koutsoyiannis (2004). Applications to rainfall maxima in the Netherlands are given by Buishand (1991) and Smits et al. (2004). The GEV cumulative distribution function $F(x)$ is given by (Jenkinson, 1955):

$$F(x) = \exp \left\{ - \left[ 1 - \frac{\kappa}{\xi} (x - \mu) \right]^{1/\kappa} \right\} \text{ for } \kappa \neq 0$$

(F1)
where

\[ y = -\ln(-\ln F) \]

The GEV distribution combines the three asymptotic extreme value distributions into a single distribution. The type of extreme value distribution is determined by

- \( \kappa = 0 \): EV1 (Gumbel distribution)
- \( \kappa > 0 \): EV2 (Fréchet type)
- \( \kappa < 0 \): EV3 (Weibull type)

The Fréchet type has a longer upper tail than the Gumbel distribution and the Weibull type a shorter tail. Using L-moment diagrams (Schaefer, 1990), Alila (1999) and Kysely and Picek (2007) show that the GEV distribution describes the distribution of the annual maximum rainfall amounts much better than the Pearson type III distribution and that the GEV distribution is generally also preferable to the generalized logistic distribution. Besides, the GEV distribution is based on asymptotic theory about the distribution of maxima.

The quantile function, the inverse of Eqs. (1) and (2), is given by:

\[ x(T) = \mu + \frac{z(1 - [-\ln(1 - T^{-1})]^{\kappa})}{\kappa} \]

\[ x(T) = \mu - \ln[-\ln(1 - T^{-1})] = \mu + sy \quad \text{for} \quad \kappa = 0 \]

where \( T = 1/(1 - F) \) is the return period.

Running annual maxima are abstracted for each of the 12 time series from the selected stations for durations \( D \) of 1, 2, 4, 8, 12 and 24 h. Running implies here that the \( D \)-hour rainfall amounts are calculated for each clock-hour of the year. A GEV distribution is fitted to the annual maxima for each station and duration separately.

Both L-moments (Hosking and Wallis, 1997) and maximum likelihood (Coles, 2001) have been used to fit the GEV distribution to annual maxima. For small samples the estimates based on L-moments generally have lower standard deviation than those based on maximum likelihood if \(-0.2 < \kappa < 0.2\) (Hosking et al., 1985). Use of maximum likelihood with a Bayesian prior distribution for \( \gamma \) (Martins and Stedinger, 2000) or with a penalty function (Coles and Dixon, 1999) performs equally well, but is computationally more difficult. Because of this and the fact that 11 stations have a record length shorter than 50 years, the method of L-moments is chosen. L-moments are based on linear combinations of the order statistics of the annual maximum rainfall amounts. First, the probability weighted moments are estimated by:

\[ b_0 = n^{-1} \sum_{j=1}^{n} x_{j,1} \]

\[ b_1 = n^{-1} \sum_{j=1}^{n} \frac{j-1}{n-1} x_{j,1} \]

\[ b_2 = n^{-1} \sum_{j=1}^{n} \frac{(j-1)(j-2)}{(n-1)(n-2)} x_{j,1} \]

where \( x_{1,1} \leq x_{2,1} \leq \ldots \leq x_{n,1} \) is the ordered sample of annual maxima. The sample L-moments are then obtained as:

\[ \ell_1 = b_0 \]

\[ \ell_2 = 2b_1 - b_0 \]

\[ \ell_3 = 6b_2 - 6b_1 + b_0 \]

The estimate \( \hat{\kappa} \) of the shape parameter \( \kappa \) follows from:

\[ \hat{\kappa} = 7.8590 c + 2.9554 c^2 \]

where

\[ c = \frac{2}{3 + \ell_3/\ell_2 - \ln 2} \]

The estimates \( \hat{\gamma} \) and \( \hat{\mu} \) of \( \gamma \) and \( \mu \) are subsequently obtained as:

\[ \hat{\gamma} = \frac{\ell_3}{(1 - 2^{-\kappa}) \Gamma(1 + \kappa)} \]

\[ \hat{\mu} = \ell_1 - \hat{\gamma} \frac{1 - \Gamma(1 + \kappa)}{\hat{\kappa}} \]

with \( \Gamma(.) \) the gamma function.

In this study \( \hat{\gamma} = \hat{\mu}/\hat{\mu} \) is considered instead of \( \hat{\gamma} \). The advantage of using \( \hat{\gamma} \) is that its correlation with \( \hat{\mu} \) and \( \hat{\kappa} \) is weak. The shape parameter \( \kappa \) is often assumed to be constant over a region and can then be estimated by combining all station records in that region. For the index-flood method \( \hat{\gamma} \) is also considered to be constant in a region. This assumption has often been made in rainfall frequency analysis (Gellens, 2002; Fowler and Kilsby, 2003; Mora et al., 2005).

Regional variability in GEV parameters

In this section the equality of GEV parameters is tested. The tests below assume that spatial dependence of the annual maxima can be neglected. Fig. 3, which is representative of all six durations, shows that the cross correlations between the annual maxima of the 12 stations are small. Data for the common period 1977–2005 were used to estimate these cross correlations. For annual maxima of daily rainfall it is further shown by Buishand (1984), using data from 140 stations located in the Netherlands, that the degree of association decreases with event magnitude. There is almost no association between the occurrence of large values if the interstation distance is larger than 30 km.

Let \( \theta_i \) be the value of a GEV parameter (\( \mu, \gamma \) or \( \kappa \)) at station \( i \). The equality of the \( \theta_i \)'s can be tested with the statistic:

\[ \chi^2 = \sum_{i=1}^{12} \left( \hat{\theta}_i - \hat{\theta}_w \right)^2 / \sigma^2(\hat{\theta}_i) \]

with \( \hat{\theta}_i \) the L-moment estimate of \( \theta_i \) and \( \hat{\theta}_w \) the weighted average of the \( \theta_i \)'s defined as:

\[ \hat{\theta}_w = \frac{\sum_{i=1}^{12} n_i \hat{\theta}_i}{\sum_{i=1}^{12} n_i} \]

where \( n_i \) is the record length at station \( i \). The variance \( \sigma^2(\hat{\theta}_i) \) in Eq. (15) was based on the asymptotic covariance matrix of the L-moment estimators of the GEV parameters as given by Hosking et al. (1985). The variance of \( \hat{\gamma} \) was obtained from the variances and covariance of \( \hat{\gamma} \) and \( \hat{\mu} \) using the delta method (Efron and Tibshirani, 1993; Coles, 2001):
of the GEV parameters. From Table 2 it can be seen that the values been verified in a Monte Carlo experiment with constant tween the annual maxima. The asymptotic distribution has degrees of freedom, if there is no spatial dependence be-

\[
\var \hat{\gamma} \approx \var \hat{\gamma} \var \hat{\mu} - 2\var \text{cov}(\hat{\gamma}, \hat{\mu}) / \mu^2
\]  

(17)

The unknown population parameters in the expressions for the variances were replaced by the weighted average \( \hat{\theta}_w \) of the at-site estimates. \( X^2 \) was calculated for \( D = 1, 2, 4, 8, 12 \) and 24 h. \( X^2 \) has an asymptotic chi-square distribution under the null hypothesis \( \theta_1 = \theta_2 = \ldots = \theta_{12} \) with 11 degrees of freedom, if there is no spatial dependence between the annual maxima. The asymptotic distribution has been verified in a Monte Carlo experiment with constant GEV parameters. From Table 2 it can be seen that the values of the \( X^2 \)-statistic vary between 5.12 and 12.64, which is far below the critical value 19.68 for a test at the 5% level.

For each duration \( D \) the \( \hat{\theta}_j \)'s were also regressed on mean annual rainfall using weighted least squares (weights proportional to \( n_j \)). Only for the location parameter of the 8- and 12-h annual maxima, the slope of the regression line was significant at the 5% level (Student's \( t \)-test, one-sided for \( \mu \), two-sided for \( \gamma \) and \( \kappa \)).

Since no geographical variation in the GEV parameters could be found and the spatial dependence between the stations' annual maxima is small, the time series from the 12 stations are concatenated to a single record of 514 years according to the station-year method.

\[
\begin{array}{cccc}
D (h) & \mu & \gamma & \kappa \\
1 & 9.82 & 10.32 & 8.82 \\
2 & 7.45 & 9.44 & 10.18 \\
4 & 5.51 & 8.98 & 10.93 \\
8 & 7.83 & 7.07 & 11.61 \\
12 & 9.35 & 7.36 & 9.16 \\
24 & 12.64 & 9.01 & 5.12 \\
\end{array}
\]

Table 2 Values of the statistic \( X^2 \) for testing equality of the GEV parameters \( \mu, \gamma \) and \( \kappa \)

Regional estimation and modelling of GEV parameters

Estimated GEV parameters for individual durations

For the time series of 514 years a GEV distribution (Eq. (1)) was fitted to the running annual maxima for durations of 1, 2, 4, 8, 12 and 24 h separately. Fig. 4 shows that the GEV distribution gives a good fit for the 1-h annual maxima and that there is a weak tendency to overestimate large quantiles of the 24-h annual maxima. A GEV distribution fitted to 514 annual maxima should give a rather good estimate of large quantiles, particularly because \( \var \gamma \) is strongly reduced.

Running annual maxima, based on clock-hour rainfall sums, tend to be smaller than sliding annual maxima, defined as maxima obtained from continuous recording. Because clock-hour sums are used, the underestimation is small for durations of 4–24 h, however a conversion has to be applied for 1- and 2-h maxima. The sliding and running annual maxima of the 84-year record of De Bilt 1906–1990 (1945 excluded) were used for this conversion. For \( \mu \) and \( \gamma \) the estimates \( \hat{\mu}(D, 514) \) and \( \hat{\gamma}(D, 514) \) from the 514-year record were multiplied by the ratio of their estimates from the sliding and running annual maxima in the 84-year De Bilt record:

\[
\begin{align*}
\hat{\mu}_s(D, 514) &= \frac{\hat{\mu}_s(D, 84)}{\hat{\mu}(D, 84)} \hat{\mu}(D, 514) \text{ for } D = 1, 2 \\
\hat{\gamma}_s(D, 514) &= \frac{\hat{\gamma}_s(D, 84)}{\hat{\gamma}(D, 84)} \hat{\gamma}(D, 514) \text{ for } D = 1, 2
\end{align*}
\]  

(18)

(19)

Parameter estimates with subscript sl refer to sliding annual maxima, the other estimates to running annual maxima. For the shape parameter \( \kappa \) no conversion was applied:

\[
\hat{\kappa}_s(D, 514) = \kappa(D, 514) \text{ for } D = 1, 2
\]  

(20)

Eqs. (18) and (19) involve three separate GEV fits. Assuming a constant \( \kappa \) in these fits did not result in a satisfactory
Rainfall depth-duration-frequency curves and their uncertainties

Figure 4  Gumbel probability plots with the GEV distribution fitted to annual 1-h (left) and 24-h (right) maxima. Dots are ordered annual maxima plotted with the Gringorten plotting position; lines represent GEV fits.

reduction of the standard deviations of $\hat{\mu}_\nu(D,514)$ and $\hat{\sigma}_\nu(D,514)$. For $\gamma_\nu(1,514)$ there was even a small increase in standard deviation due to a change of sign of the correlation between $\gamma_\nu(1,84)/\gamma(1,84)$ and $\gamma(1,514)$.

For $\hat{\mu}(D,514)$ a conversion factor of 1.13 was found for $D = 1$ and 1.04 for $D = 2$. The value of 1.13 for $D = 1$ corresponds quite well with the correction factors (known as the Hershfield factor) 1.15 in the UK Flood Studies Report (NERC, 1975) and 1.13 in Hershfield (1961) for quantiles of clock-hour maxima. The conversion factors for $\gamma(1,514)$ were 0.94 and 0.98 for $D = 1$ and $D = 2$, respectively. This implies that the correction factor for quantiles decreases with increasing return period. A disadvantage of the conversion of $\gamma(1,514)$ is that it leads to a considerable increase in the standard deviation (see below).

Table 3 gives the estimated GEV parameters and their standard deviations. As expected $\hat{\mu}$ increases with increasing $D$. The parameter $\gamma$ increases with decreasing duration. For this parameter the standard deviation is relatively high for $D = 1$ and 2 h as a result of the use of a short record to adjust the estimate from the 514-year record. There seems to be no systematic variation of $\kappa$ with $D$. This is in line with results of Gellens (2003) for Belgium. The values of $\kappa$ deviate 3.5 to 4 times their standard deviation from 0, so the Gumbel distribution would not be appropriate in modelling the annual rainfall maxima. Negative values of $\kappa$ have been found in many other studies for $D < 24$ h. E.g. Koutsoyiannis (2004) observed that $\kappa = -0.15$ for daily annual maximum rainfall in different climatic zones of the USA, the UK and the Mediterranean.

In contrast to the use of asymptotic expressions as in Section "Regional variability in GEV parameters", the standard deviations in Table 3 were derived from the bootstrap. In the bootstrap method new samples (bootstrap samples) are generated by sampling with replacement from the original sample (Diaconis and Efron, 1983; Efron and Tibshirani, 1989) were the first who used the bootstrap to determine the uncertainty of design storms.

Correlations of estimated GEV parameters

The bootstrap also provides for each GEV parameter the correlation coefficients between the estimates for different durations, which are needed for the assessment of the change of the parameter with duration. Table 4 shows correlation matrices of $\hat{\mu}$, $\hat{\gamma}$, and $\hat{\kappa}$ based on the same $10^4$ bootstrap samples as in Section "Estimated GEV parameters for individual durations". Each correlation matrix consists of the correlations between the parameter estimates for different durations. These correlations are due to the dependence between the annual rainfall maxima for different durations. As a result correlations between neighbouring durations are quite large. Correlation coefficients for $\hat{\kappa}$ and $\hat{\gamma}$ are in general lower than for $\hat{\mu}$. Table 4 also provides the correlations between the estimates of different GEV

<table>
<thead>
<tr>
<th>$D$ (h)</th>
<th>$\hat{\mu}$ (mm)</th>
<th>$\hat{\gamma}$</th>
<th>$\hat{\kappa}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14.04 (0.34)</td>
<td>0.343 (0.019)</td>
<td>-0.127 (0.033)</td>
</tr>
<tr>
<td>2</td>
<td>16.79 (0.31)</td>
<td>0.325 (0.016)</td>
<td>-0.112 (0.032)</td>
</tr>
<tr>
<td>4</td>
<td>20.08 (0.30)</td>
<td>0.300 (0.011)</td>
<td>-0.102 (0.029)</td>
</tr>
<tr>
<td>8</td>
<td>24.27 (0.33)</td>
<td>0.271 (0.010)</td>
<td>-0.132 (0.033)</td>
</tr>
<tr>
<td>12</td>
<td>29.31 (0.36)</td>
<td>0.268 (0.010)</td>
<td>-0.121 (0.033)</td>
</tr>
<tr>
<td>24</td>
<td>33.08 (0.43)</td>
<td>0.253 (0.010)</td>
<td>-0.117 (0.030)</td>
</tr>
</tbody>
</table>

Standard deviations are estimated with the bootstrap and given between brackets.
parameters for the same duration and shows that especially $\text{corr}(\hat{\mu}, \hat{\gamma})$ and $\text{corr}(\hat{\kappa}, \hat{\gamma})$ are rather small.

**GEV parameters as a function of duration**

Relations of GEV parameters as a function of duration $D$ (hour) are used to construct rainfall DDF curves. In Fig. 5 GEV parameters are plotted against $D$ for $1$, $2$, $4$, $8$, $12$, and $24$ h. For $D = 1$, $2$ h GEV parameters are calculated with Eqs. (18)--(20). It is shown that $\gamma$ and the logarithm of $\mu$ have a linear relationship with the logarithm of $D$ for $D = 1$--$24$ h. There appears to be no systematic variation of $\kappa$ with duration.

Fig. 5 suggests the following regression model for the GEV parameters:

$$\hat{\theta} = X\hat{\beta} + \epsilon$$

with $\hat{\theta}$ the vector containing the estimated values of the GEV parameter $\theta$ (or its logarithm) for the six durations $D_1, \ldots, D_6$,

$$X = \begin{pmatrix} 1 & \ln D_1 \\ \vdots & \vdots \\ 1 & \ln D_6 \end{pmatrix},$$

$$\hat{\beta} = \begin{pmatrix} a_1 \\ b_1 \end{pmatrix},$$

and $\epsilon$ a vector of random disturbances. The components of $\epsilon$ are correlated because of the correlation between the estimated GEV parameters for different durations. To account for this correlation, the method of generalized least squares was chosen to estimate the regression coefficients $a$ and $b$.

The generalized least squares estimate $\hat{\beta}_{\text{GLS}}$ of $\beta$ is given by:

$$\hat{\beta}_{\text{GLS}} = (X^TC^{-1}X)^{-1}X^TC^{-1}\hat{\theta} = \begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix}$$

with $C$ the covariance matrix of $\theta$. For $\hat{\gamma}$ and $\hat{\kappa}$ the estimated correlation coefficients are given in Table 4; for $\ln \hat{\mu}$ the estimated correlation coefficients are almost the same as those for $\hat{\mu}$. Because of the long time series, these correlation estimates are relatively accurate. Stedinger and Tasker (1985) compared generalized least squares regression with ordinary and weighted least squares in a regional analysis of streamflow statistics. Madsen et al. (2002) applied generalized least squares in a regional frequency analysis of rainfall to account for intersite dependence. The covariance matrix of $\hat{\beta}_{\text{GLS}}$ is given by:

$$\text{cov}(\hat{\beta}_{\text{GLS}}) = (X^TC^{-1}X)^{-1}$$

This can be compared with the covariance matrix of the ordinary least squares estimate $\hat{\beta}_{\text{OLS}}$:

$$\text{cov}(\hat{\beta}_{\text{OLS}}) = (X^TX)^{-1}X^TCX(X^TX)^{-1}$$

The goodness-of-fit can be tested with the statistic:

$$\chi^2 = (\hat{\theta} - \hat{\theta}_{\text{OLS}})^TC^{-1}(\hat{\theta} - \hat{\theta}_{\text{OLS}})$$

where $\hat{\theta}_{\text{GLS}} = X\hat{\beta}_{\text{GLS}}$. In the case of an adequate fit, $\chi^2$ is approximately chi-square distributed with $n - p$ degrees of freedom, with $n = 6$ the number of durations and $p$ the number of regression coefficients. Eq. (25) generalizes Eq. (15) in two directions: it allows for dependence between the estimated GEV parameters, and it allows for the inclusion of covariates.

Table 5 shows the estimated regression coefficients with their standard deviations. For $\kappa$ the estimate of the slope $b$ differs no more from zero than $0.46\sigma(b)$. This confirms that $\kappa$ may be considered to be constant. Re-estimating $a$ yielded $\hat{\kappa}_{\text{GLS}} = \hat{a} = -0.114$, which is given by the solid line in Fig. 5. For the other two GEV parameters there is a significant dependence on duration. The values of the $\chi^2$-statistic in Table 5 are well below the critical values for a test at the 5% level. So there is no evidence of lack-of-fit.

For $\sigma(\hat{\kappa}_{\text{GLS}})$ a value of 0.021 was found, which is considerably smaller than the standard deviation of the $\hat{\kappa}$’s for the individual durations in Table 3. The ordinary least squares estimate of $\kappa$ is simply the average $-0.119$ of the six estimates for the individual durations. The standard deviation of this average, 0.023, is somewhat larger than $\sigma(\hat{\kappa}_{\text{GLS}})$. This confirms the conclusion in Buishand (1993) that

| Table 4: Correlation matrices of $\hat{\mu}$, $\hat{\gamma}$ and $\hat{\kappa}$ for $D$-hour annual maximum rainfall depths and correlations between the estimates of different GEV parameters for the same duration, both estimated with the bootstrap |
|-----------------|-----------------|-----------------|
| $D$ | $\hat{\mu}$ | $\hat{\gamma}$ | $\hat{\kappa}$ |
| 1 | 1.00 | 1.00 | 1.00 |
| 2 | 0.52 | 1.00 | 0.49 | 1.00 |
| 4 | 0.47 | 0.79 | 0.39 | 0.62 |
| 8 | 0.36 | 0.62 | 0.33 | 0.41 |
| 12 | 0.30 | 0.52 | 0.22 | 0.35 |
| 24 | 0.20 | 0.40 | 0.15 | 0.23 |

For $\hat{\mu}$, $\hat{\gamma}$, and $\hat{\kappa}$, the estimated correlation coefficients are given in Table 4; for $\ln \hat{\mu}$, the estimated correlation coefficients are almost the same as those for $\hat{\mu}$. Because of the long time series, these correlation estimates are relatively accurate. Stedinger and Tasker (1985) compared generalized least squares regression with ordinary and weighted least squares in a regional analysis of streamflow statistics. Madsen et al. (2002) applied generalized least squares in a regional frequency analysis of rainfall to account for intersite dependence. The covariance matrix of $\hat{\beta}_{\text{GLS}}$ is given by:

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For $\sigma(\hat{\kappa}_{\text{GLS}})$ a value of 0.021 was found, which is considerably smaller than the standard deviation of the $\hat{\kappa}$’s for the individual durations in Table 3. The ordinary least squares estimate of $\kappa$ is simply the average $-0.119$ of the six estimates for the individual durations. The standard deviation of this average, 0.023, is somewhat larger than $\sigma(\hat{\kappa}_{\text{GLS}})$. This confirms the conclusion in Buishand (1993) that...
the reduction in standard deviation due to the use of generalized least squares is small in the case of rainfall depth-duration-frequency analysis. The correlation between the estimated GEV parameters may, however, not be ignored if standard deviations or goodness-of-fit are of interest.

The systematic change of $c$ with duration implies that the dependence of quantiles on $T$ and $D$ cannot be separated in a function of $D$ and a function of $T$, as was done in Koutsoyiannis et al. (1998). Another consequence is that simple scaling does not apply as observed by Menabde et al. (1999) for Melbourne (Australia) and Warmbaths (South Africa) and Borga et al. (2005) for the Trentino province (Italy). Both require that the coefficient of variation is constant, which is not the case if $c$ varies with duration.

Construction of rainfall DDF curves and their uncertainties

Derivation of DDF curves

Now that the GEV parameters are described as a function of $D$, rainfall DDF curves are constructed by substituting these relationships into Eq. (3), so that the DDF curves are given by:

$$
\hat{x}(T) = \exp(\hat{a}_l + \hat{b}_l \ln D) \\
\times \left(1 + (\hat{a}_c + \hat{b}_c \ln D)^\left(1 - \frac{\ln(1 - T^{-1})^{\hat{k}_{\text{GLS}}}}{\hat{k}_{\text{GLS}}}ight)\right)
$$

(26)

where $\hat{a}_l = 2.629$, $\hat{b}_l = 0.273$, $\hat{a}_c = 0.341$, $\hat{b}_c = -0.028$ and $\hat{k}_{\text{GLS}} = -0.114$.

By choosing a return period $T$, the rainfall depth $x$ (mm) can be plotted as a function of duration $D$ using Eq. (26). In Section "GEV parameters as a function of duration" it was noticed that $\sigma(\bar{k})$ is considerably reduced if $\bar{k}$ is based on the maxima for all six durations. Large quantiles are then more accurately estimated compared to fitting a GEV distribution only to the maxima for the duration of interest. Fig. 6 presents the DDF curves for $T = 100$ and 1000 year. The curves show a strong increase of $x$ with $D$, e.g. for $T = 1000$ year rainfall depths range from 64 to 120 mm for $D = 1-24$ h.

Modelling uncertainty in DDF curves

Uncertainty in DDF curves is usually disregarded, while it should be considered, e.g. in the design of hydraulic structures. Here the bootstrap was applied to assess this uncertainty. This method considers only the uncertainty due to the estimation of the GEV parameters, i.e. sampling errors. For each of the $10^4$ bootstrap samples from Section "Estimated GEV parameters for individual durations" the relations between the GEV parameters and duration were

| Table 5 Results of the regression of GEV parameters |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| GEV parameter   | $\hat{a}$       | $\sigma(\hat{a})$ | $\hat{b}$       | $\sigma(\hat{b})$ | $\chi^2$       |
| $\ln\mu$        | 2.629           | 0.019           | 0.273           | 0.006           | 3.10           |
| $\gamma$        | 0.341           | 0.015           | -0.028          | 0.005           | 4.07           |
| $\kappa$        | -0.125          | 0.032           | 0.006           | 0.013           | 2.38           |
| $\kappa_{\text{GLS}}$ | -0.114       | 0.021           | -              | -               | 2.60           |

Figure 5 GEV parameters plotted against duration $D$. The solid lines represent the generalized least squares fits to the estimated GEV parameters.

Figure 6 Rainfall DDF curves (solid lines) and 95%-confidence bands (dashed lines) for return periods of 100 and 1000 year (indicated on the right axis).
re-estimated using generalized least squares, so that $10^4$ DDF curves could be constructed. For each DDF curve the rainfall depths were derived for durations between 1 and 24 h in steps of 1 min. Subsequently, for each of these durations the $10^4$ depths were ranked in increasing order and the 250th and 9750th values were determined. Next, these values were plotted and formed the 95%-confidence bands. For return periods of 100 and 1000 year, Fig. 6 shows the DDF curves and their 95%-confidence bands. For $T = 1000$ year the confidence interval ranges from 55–73 mm for $D = 1$ h to 106–135 mm for $D = 24$ h, thus showing a rather large uncertainty, despite the fact that the DDF curves are based on a 514-year record. For longer $T$ uncertainty increases substantially.

It is found that the bootstrap distribution of the estimated quantiles can be described by a lognormal distribution. The parameters $\xi$ and $\sigma$ of the lognormal distribution, i.e. the mean and standard deviation of the underlying normal distribution, are modelled as a function of $D$ and $T$. The parameter $\xi$ is well described by the natural logarithm of the estimated quantiles from the DDF curve. Using $10^4$ bootstrap samples for six durations and seven return periods, namely from every combination of $D = 1$, 2, 4, 8, 12 and 24 h and $T = 10$, 20, 50, 100, 200, 500 and 1000 year, the parameter $\sigma$ is modelled as:

$$\sigma = -0.0042 + 0.0103 D^{-1} + 0.0091 \ln T$$

The regression coefficients were estimated with ordinary least squares. Fig. 7 shows the DDF curves for $T = 100$ and 1000 year. The lognormal probability density functions which describe the uncertainties in these curves are plotted for $D = 1$ and 12 h.

Conclusions

Extreme rainfall in the Netherlands for durations between 1 h and 24 h was studied. Since regional variability in extreme rainfall statistics could not be found and spatial dependence between extreme rainfall amounts appeared to be small, a record of 514 annual maxima was constructed according to the station-year method. GEV parameters of this time series were estimated with the method of L-moments. Standard deviations and correlations of estimated GEV parameters were obtained with the bootstrap. To take into account the correlation between estimated GEV parameters for different durations, the generalized least squares method was used to describe the variation of these parameters as a function of duration. The relations were used to construct rainfall DDF curves. Finally, uncertainties in DDF curves, due to sampling variability, were quantified with the bootstrap and described with a lognormal distribution.

It was found that the shape parameter $\kappa$ of the GEV distribution does not change with duration. For the parameter $\gamma$ there is a significant increase with decreasing duration. As a consequence, the coefficient of variation increases with decreasing duration. This implies that simple scaling does not hold.

A 84-year record of sliding 1- and 2-h annual maxima from De Bilt was used to convert the estimated GEV parameters from the 514-year hourly station-year record. For the parameter $\gamma$ a correction factor $<1$ was found which implies that the correction for quantiles depends on return period. This may be related to the change of $\gamma$ with duration.

In the generalized least squares method used in this study to fit relationships between GEV parameters and duration, the fit for one GEV parameter does not affect the fit for the other two parameters. This may be justified by the fact that the correlation between the estimated GEV parameters is small. As an alternative, re-estimation of the parameters $\mu$ and $\gamma$ in the 514-year record was explored assuming $\kappa = \kappa_{\text{GLS}}$. The changes were very small.

Possible inhomogeneities in the rainfall records due to changed measurement methods, gauge types or locations were not considered. Further, annual maxima were assumed to be stationary. Smits et al. (2004) found for De Bilt that the trends in the extremes are relatively small.

Usually uncertainties in rainfall DDF curves are disregarded, so that risks can be underestimated. An innovative aspect of this study is that uncertainty in DDF curves due to sampling variability is taken into account. Other sources of uncertainty were not considered, such as measurement errors and uncertainty about the choice of the distribution.

A situation that there is no regional variability in all GEV parameters will seldom be met in other parts of the world. In addition, there might be spatial correlation between annual maximum rainfalls. The methods used here can be adapted if regional variability in GEV parameters is present. Resampling from the $N$ years providing rainfall data, as e.g. in Faulkner and Jones (1999), rather than resampling from all station-years, should be considered. Some care is needed in situations where the covariance matrices of the estimated GEV parameters vary over the region of interest.

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Appendix A. Bootstrap algorithm for estimating standard deviations and correlation coefficients of estimated GEV parameters

The bootstrap is applied to the running and sliding annual maximum rainfalls. Because the sliding annual maxima are only available for De Bilt, the annual maxima from De Bilt and the other stations are sampled separately. The two bootstrap samples are then concatenated to form a bootstrap sample of 514 years with the same layout as the original sample of annual maximum rainfalls. This procedure is similar to the regional bootstrap in GREHYS (1996). A precise description of the bootstrap algorithm is presented below. Steps 3 and 4 are illustrated in Table A.1.

1. Split the annual maxima series of 514 years into a series of 84 years with year numbers 1, 2, ..., 84, belonging to De Bilt 1906–1990, and a series of 430 years with year numbers 85, 86, ..., 514.
2. Draw for each of the two series of year numbers a random sample with replacement (bootstrap sample).
3. Select the running annual maxima for the sampled year numbers for D = 1, 2, 4, 8, 12, 24 h. This leads to one bootstrap sample of 84 years of running annual maxima and one bootstrap sample of 430 years of running annual maxima for each D. For D = 1, 2 h also a bootstrap sample of 84 sliding annual maxima is constructed using the same year numbers as for the 84 running annual maxima.
4. Construct bootstrap samples of 514 years by adding the 84-year to the 430-year running annual maxima.
5. Fit a GEV distribution to the bootstrap sample of 514 years for each of the durations of 1, 2, 4, 8, 12 and 24 h.
6. Fit a GEV distribution to the bootstrap sample of 84 years for durations of 1 and 2 h individually. Do this separately for the bootstrap samples of running and sliding maxima.
7. Apply Eqs. (18)–(20) for D = 1, 2 h.
8. Repeat this 10⁴ times, so that 10⁴ GEV parameters are estimated for each duration.
9. Determine the standard deviation of each estimated GEV parameter as the sample standard deviation of that parameter estimate in the 10⁴ bootstrap samples.
10. Determine the correlation between estimated GEV parameters by calculating the correlation between these parameter estimates in the 10⁴ bootstrap samples.

References