S1 EBM simulations with linear surface albedo feedback

The results of EBM simulations with linear surface albedo are presented here. They confirm the interpretation that spatially confined linear feedbacks (rather than nonlinear feedbacks) interacting with a nonlocal representation of energy transport is the underlying reason that the top-of-atmosphere energy balance analysis for regional surface temperature changes does not account for the EBM simulations with locked surface albedo or energy transport.

For the radiatively forced perturbation simulations, the surface albedo depends linearly on the surface temperature change in a limited latitudinal range:

\[
\alpha_{\text{linear}}(T_s, \phi) = \begin{cases} 
\alpha_{\text{REF}}(\phi) + \gamma [T_s(\phi) - T_s|_{\text{REF}}(\phi)] & \text{if } 50^\circ < \phi < 65^\circ \\
\alpha_{\text{REF}}(\phi) & \text{elsewhere ,}
\end{cases}
\]  

(S1)

where \( \gamma = -0.02 \, ^\circ \text{C}^{-1} \) and \( \alpha_o \) is the minimum albedo allowed. Similar results can be obtained if a minimum albedo of 0 is used. The reference albedo and surface temperature about which the perturbation albedo is linearized, \( \alpha_{\text{REF}} \) and \( T_s|_{\text{REF}} \), are taken from the control simulation, which uses \( \alpha(T_s) \) given by (4) of the main text. The latitudinal range to which the feedback is confined in (S1) is consistent with the region of the control simulation where the albedo transitions from \( \alpha_o \) to \( \alpha_i \) (Fig. S1b). All other energy balance model parameters follow those presented in the main text.

S2 Local and global surface albedo feedback definitions

Figure S3 shows the latitudinal structure of the surface albedo feedback for the simulations with the nonlinear temperature-dependent albedo presented in the main text. The red lines in Fig. S3 are identical to those in Fig. 1c. The magenta lines show the alternative definition of albedo feedback that uses the global-mean surface temperature change. Independent of the surface temperature used to define the feedback, the simulations with locked energy transport have a substantially different magnitude surface albedo feedback. However, the locked energy transport simulation has a larger surface albedo feedback than the full EBM using the global-mean surface temperature change definition, which is the opposite from the local surface temperature change definition.
Figure S1: (a) Surface temperature and (b) surface albedo in the control (black) and perturbation (red) simulations for three EBM variants (indicated in the legend) with linear surface albedo feedback (S1). (c) Surface albedo feedback for three EBM variants, where the feedback has been normalized by the local surface temperature change: $\frac{1}{4}Q S(\phi) \Delta \alpha(\phi) \times [\Delta T_s(\phi)]^{-1}$.
Figure S2: (a) Surface temperature change and components of the TOA energy balance surface temperature analysis (5) of the “full EBM” with interactive surface albedo and interactive atmospheric energy transport (colored dashed lines are surface temperature changes in °C associated with individual components of the TOA energy balance, as indicated in the legend, and the gray dashed line is their sum) in the EBM simulation with linear surface albedo feedback (S1). Surface temperature change (black) and expected surface temperature change from TOA energy balance (gray) in locked simulations with (b) prescribed surface albedo and (c) prescribed atmospheric energy transport with in EBM simulations with linear surface albedo feedback (S1). In panels b and c, the change in individual components of the TOA energy balance are taken from the full EBM with linear surface albedo temperature dependence.
Figure S3: Surface albedo feedback for three EBM variants using the nonlinear temperature-dependent albedo (4) of the main text. Red lines are feedbacks defined using the local surface temperature change as in the main text: \(\frac{1}{4} QS(\phi) \Delta a(\phi) \times |\Delta T_s(\phi)|^{-1}\). Magenta lines are feedbacks defined using the global-mean surface temperature change: \(\frac{1}{4} QS(\phi) \Delta a(\phi) \times \langle \Delta T_s \rangle^{-1}\), with \(\langle \cdot \rangle\) indicating the global mean.